

# **A report of Research-Based Workshop on Teaching Fractions and Ratios**

**CESME**

(under PMMMNMTT scheme of MHRD)

**Date 7th to 18th June 2021**



**Homi Bhabha Centre for Science Education  
Tata Institute of Fundamental Research  
June 2021**

## Introduction

As part of the Centre of Excellence in Science and Mathematics Education (CESME, HBCSE), Mathematics Education group conducted a teacher development workshop for middle school Marathi medium teachers. It was conducted in online mode.

Most of the research material from across the countries that describes teaching fractions and ratios is primarily available in English. Focus of this workshop was to give the participant teachers an exposure to the research done by several researchers worldwide and at HBCSE on teaching fractions and ratios with understanding. While working with teachers, it was realised that teachers need to engage with the products of research in a practice-based way. This workshop is an attempt to develop such resources for the teachers and engage teachers in those with two lenses - student thinking and reasoning and making use of sub-construct theory of fractions to navigate middle grades curriculum.

## Brief Information about Each Day:

1. Day 1 (7 June 2021): This session was an introductory session where participants introduced themselves; briefly shared their thoughts about fractions teaching and the reason for enrolling on the course. In this session, we gave a brief introduction to educational research work and different projects of HBCSE. The following course aims were discussed: understanding and appreciating the importance of deep knowledge of fractions for teaching; how children learn fractions; tasks with learner-centred pedagogy; developing skills towards teaching fractions by analysing fractions' teaching; developing effective assessment. Towards the end, participants discussed a class scenario that connected fractions with the division-remainder.
2. Day 2 (8 June 2021): This was a theory-based session and focused on five different meanings of fractions. These meanings, referred to as sub-constructs of fractions, are as follows: part of a whole, operator, ratio, quotient, and measure. We discussed what each sub-construct means, its examples and their curricular presence. Teachers worked on examples of each.
3. Day 3 (9 June 2021): The session began by understanding connections across five sub-constructs and their relationship with rational numbers. The primary focus of the session was to distinguish multiplicative reasoning in contrast to additive reasoning. To achieve this, we discussed various examples, and participants also enacted an exercise of

constructing structures. The session ended with refining the meanings of operations such as multiplication and division within different number domains.

4. Day 4 (10 June 2021): In the beginning the session focused on understanding direct and inverse proportion through multiplicative thinking, further there was part of the session which was focused on understanding unit fractions and comparison of fractions through measure sub-construct and by using that how one can compare fractions. In the last part the instructor discussed comparing fractions through whole part sub-construct and quotient subconstruct.
5. Day 5 (11 June 2021): This session focused on comparison of fractions and then on understanding fractions through context of sharing. Then the participants analysed students' responses around unit fractions and on exercises that involved finding fractions between two fractions. Session concluded with discussion on equivalence class.
6. Day 6 (14 June 2021): This session focused on making and using Teaching Learning Material (TLM) for fractions. The instructor demonstrated how to make fraction strips. He used a photographic presentation to aid the making of the fractions strips. Once the participants completed the fractions strips, the instructor used the strips for different activities/operations of fractions such as comparison, equivalence and addition. Some participants also shared their ideas for this TLM. Towards the end, the instructor discussed some students' responses.
7. Day 7 (15 June 2021): The session began by an exercise of making word problems on multiplication and division of fractions. The teachers then studied different meanings of fraction division and fraction multiplication. Then further there was a discussion about different meanings of fraction division which involves measure, equal distribution, and opposite of cartesian multiplication (assuming fractions as sides of rectangles).
8. Day 8 (16 June 2021): The session began by discussing cartesian multiplication- meaning of fraction division, we explained it through example. Then participants were exposed to Piaget's idea of constructivism and three types of knowledge as helpful in learning fractions. The three types of knowledge are physical, social, and logico-mathematical. For each type teachers worked on examples and participated in an activity called missing area.
9. Day 9 (17 June 2021): The session continued the discussion of missing area activity and continued with examples from real life on situation of ratios and proportions (vaccine efficacy study). The teachers then worked on several problems and delineated use of

additive, multiplicative and ratio reasoning. Towards the end everyone worked around a fallacy that highlighted meanings of operations in different measure spaces.

10. Day 10 (18 June 2021): The session we started with some examples of ratios and some examples of ratios. Then we discussed the teaching trajectory which involved examples from different meanings of fraction and one can use those meanings while teaching. In the end, the participant teachers shared their experiences of being part of this workshop and gave feedback.

## Some responses of teachers:

1. <sushant how are you going to manage this?>

## List of Participants:

Sr. No.	Name	Gender	Designation	Affiliation
1	Arti Rajkumar Thakur	F	Teacher	Chhatrapati Shivaji Vidyalaya, Thane
2	Arun Bais	M	Teacher	DIET HINGOLI
3	Devidas Gajanan Gosavi	M	Teacher	DIET Buldana
4	Gajanan Deshmukh	M	Teacher	Z. P. School Morgaon
5	Jaimala Gawande	F	Teacher	Anand Niketan School, Sewagram, Wardha
6	Jayashree Kamde	F	Teacher	Anand Niketan School, Sewagram, Wardha
7	Lalita Bhamre	F	Teacher	Z. P. School Akkalkuva, Nandurbar
8	Manisha Kurhade	F	Teacher	Z. P. Primary School Anandwadi, Pune
9	Manjusha Kamalakar Swami	F	Teacher	Z.P.P.S.KINI, OSMANABAD
10	Pentu Maisanwad	M	Teacher	Z. P. Primary School Dahifale Bhongane, Jalna
11	Prakash Uttam Sanap	M	Teacher	Z P Primary School Manewadi, Satara
12	Rajashree	F	Teacher	Anand Niketan School, Sewagram,

	Chowdhari			Wardha
13	Ranjana Sopan Shinde	F	Teacher	Borhadewadi girls School, Moshi, Pune
14	Sandeep Wakchaure	M	Teacher	Zila Shikshan and Prashikshan, Sangamner, Ahmednagar
15	Sanjay Maroti Mathankar	M	Teacher	Zilla Parishad Upper Primary School Mangli Rai
16	Sarika Balkrishna Patil	F	Teacher	R. Z. P. Karawali wadi, Panvel
17	Satyashray Hasabnis	M	Teacher	Jnana Prabodhini Prashala, Pune
18	Sharad Taksande	M	Teacher	Anand Niketan School, Sewagram, Wardha
19	Sucharita Kaley	F	Teacher	DIET Gadchiroli
20	Suryakant Laxman Shewale	M	Teacher	R. Z. P. Pendhar
21	Tulashiram Thakare	M	Teacher	Z. P. Primary School Nave Ratir, Nashik

## List of resource persons:

Sr. No.	Name	Gender	Designation	Affiliation
1	Dr Shweta Naik	F	Scientific Officer - C	HBCSE - TIFR
2	Mr Sushant Pawar	M	Project Scientific Assistant - B	HBCSE - TIFR

## Timetable:

Session Number	Date of the session	Time of the session	Topic of the session

1	7 June 2021	16:00 - 17:30	Introduction to the Course
2	8 June 2021	16:00 - 17:30	Understanding Different Meanings of Fractions
3	9 June 2021	16:00 - 17:30	Discussion of Operator and Ratio Subconstruct, and Multiplicative thinking
4	10 June 2021	16:00 - 17:30	Understanding direct and inverse proportion, understanding unit fractions and comparison of fractions through measure subconstruct
5	11 June 2021	16:00 - 17:30	Understanding fractions through quotient sub construct, analysis of students' responses, and understanding equivalent fractions
6	14 June 2021	16:00 - 17:30	Making and Using Teaching Learning Material (TLM) For Learning Fractions and analysis of students' responses
7	15 June 2021	16:00 - 17:30	Making word problems and Understanding Meaning of Multiplication and Division Of Fractions
8	16 June 2021	16:00 - 17:30	Division of fractions, Understanding Constructivism and Different Types of Knowledge, and activity of missing area
9	17 June 2021	16:00 - 17:30	Some real life examples of proportion , analysis of questions on ratio and proportion, naming the fractions
10	18 June 2021	16:00 - 17:30	Ratio - Proportion, Teaching Trajectory, and Feedback

# Some glimpse of the Workshop

## Research Based Teaching Fractions-Ratios Workshop

### स्पष्टीकरण

एका विद्यार्थ्याने वर्गात खालीलप्रमाणे गणित सोडवले, आणि शिक्षिकेने ते चूक असे सांगितले

$$\frac{3}{7} + \frac{2}{3} = \frac{5}{10}$$

दुसऱ्या दिवशी विद्यार्थ्यांचे पालक शाळेत आले. त्यांनी शिक्षिकेला उत्तरपत्रिकेवरील त्यांचे लिखाण दाखविले, ज्यात इतिहासात 50 पैकी 33, व भूगोलात 50 पैकी 23 असे मार्क मिळाले होते. शिक्षिकेने त्याचे समाजशास्त्रात 100 पैकी 56, म्हणजे 56/100 असे मार्क दिले होते. पालक म्हणाले इथे तुम्हीही माज्या मुलाप्रमाणे जोड केली आहे. मग हे बरोबर आणि त्याचे चूक असे का?

Recorded with HBCSE Webinar.



## Research Based Teaching Fractions-Ratios Workshop

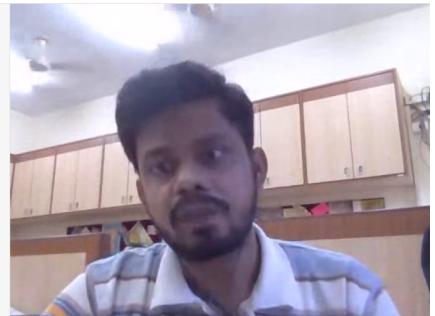
A chessboard has area  $8 \times 8 = 64$  sq units. Cut it into four pieces as shown.

Reassemble the pieces to form a rectangle. Now the area is  $5 \times 13 = 65$  sq units !

How did the area increase?  
Look carefully the slopes of the edges which meet at the joint.  
There is a thin gap where the pieces meet which has an area of exactly 1 sq unit!!  
Notice that the sides forming right angles in all the pieces have lengths equal to Fibonacci numbers.

slope =  $-\frac{2}{5}$   
slope =  $-\frac{3}{8}$

Recorded with HBCSE Webinar.



Manisha Kurhade: Good afternoon  
Sushant Pawar: 1, 2, 3, 5, 8, 13, 21, ...  
Shweta S. Naik: 1, 1  
Satyashray Shrikant Hasabnis: fibonacci sequence  
Rajashree: nahi  
Tulashiram Thakare: नमस्ते  
Rajashree: ho sir

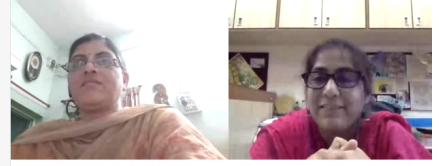


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### रचना: गणिताच्या शिक्षणातील संशोधन - अपूर्णाकांचे अध्यापन

- ♦ अध्यापनासाठीचे अपूर्णाकांचे सखोल ज्ञान (2 sessions)
- ♦ विद्यार्थी-केंद्रीत अध्यापनशास्त्र: मुले अपूर्णाक कसे शिकतात हे समजून घेणे (2 sessions)
- ♦ अपूर्णाक शिकवण्याच्या पद्धतींचे विश्लेषण (2 sessions)
- ♦ अपूर्णाकांवरील गणिती क्रियांचे विश्लेषण आणि (3 sessions)
- ♦ प्रभावी मूल्यांकन विकसित करणे (1 session)

Recorded with HBCSE Webinar.

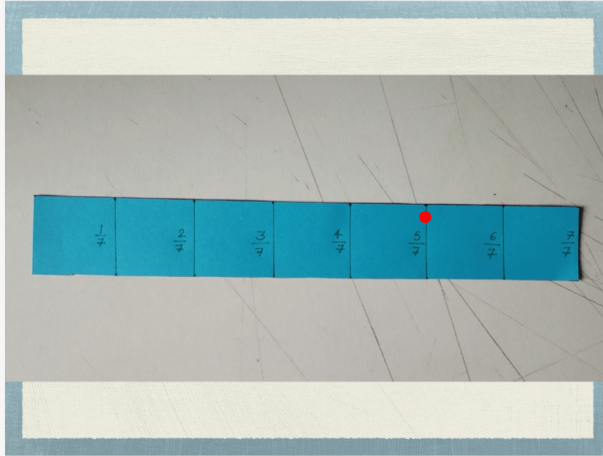


completely changed.... my interest to join your training is to reach n cater needs of my kids specially to higher primary schools..... to make fractions more friendly .

Shweta S. Naik:great!

Satyashray Hasabnis:वर्धा येथे २ वर्षा पूर्वी 'गांधी विज्ञान शिबीर' ला आलो होतो. त्यावेळी मला नई तालीम शाळेमध्ये अध्ययन आणि अध्यापन कसे घेतले जाते हे जाणून घ्यायचे होते. मात्र शाळेला सुट्टी दिलेली असल्याने ती इच्छा अपूर्ण राहिली.

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Prakash Sanap:Image disat nahi

Shweta S. Naik:yes

Shweta S. Naik:Sundar ekdam!



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### फीडबॅक आणि इतर काही प्रश्न

- ♦ जमल्यास आपला विडिओ ऑन करा

Recorded with HBCSE Webinar.



Satyashray Shrikant Hasabnis: धन्यवाद प्रकाश सर  
Shweta S. Naik: रंजना मॅडम तुमची समरी आवडली, खास करून तुम्ही आपल्या सहभागी शिक्षकांच्या ज्या skills नोंदीस केल्यात ते आवडले  
Shweta S. Naik: Thank you Prakash sir ani Ranjana Mam  
Shweta S. Naik: Rajashree mam, rahilya ka fakt?  
Shweta S. Naik: tumhi shweta hi mhanu shakata  
Sushant Pawar: Sushant ase hi bolu shakta mala

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### फीडबॅक आणि इतर काही प्रश्न

- ♦ जमल्यास आपला विडिओ ऑन करा

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Sushant Pawar: <https://mathedu.hbcse.tifr.res.in/>  
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Satyashray Shrikant Hasabnis: धन्यवाद सुरांत  
Shweta S. Naik: <https://mathedu.hbcse.tifr.res.in/>  
Shweta S. Naik: 25072114  
Devidas Gosavi: खूप इच्छा असूनही बोलता येत नाही  
Shweta S. Naik: देविदास सर आपण फोन वर बोलूयात